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[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)Palatini  $f(R)$  gravity as a fixed point

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## ABSTRACT

In the context of modified gravity, we point out how the Palatini version of these theories is singled out as a very special case corresponding to the unique fixed point of a transformation involving a special conformal rescaling of the metric. This mathematical peculiarity signals deeply rooted problems which make the theory unphysical.

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## 1. Introduction

Among the multitude of efforts devoted to explaining and modelling the current acceleration of the cosmic expansion discovered with type Ia supernovae [1], modified (or  $f(R)$ ) gravity has received much attention. This class of theories aims at disposing of the concept of dark energy by assuming that, instead, we may be observing the first deviations from Einstein's general relativity on cosmological scales ([2], see [3] for a review). Modified gravity is described by the action

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) + S^{(m)}[g_{ab}, \psi], \quad (1.1)$$

where  $S^{(m)} = \int d^4x \sqrt{-g} \mathcal{L}^{(m)}$  is the matter action and  $\psi$  collectively denotes the matter fields,  $g$  is the determinant of the space-time metric  $g_{ab}$ ,  $\kappa \equiv 8\pi G$ , where  $G$  is Newton's constant (we follow the notations of [4] and use units in which  $G = c = 1$ ), and  $f(R)$  is a (generically nonlinear and twice differentiable) function of the Ricci curvature  $R$  which generalizes the Einstein–Hilbert action, to which it reduces when  $f(R) = R$ . Modified gravity comes in three versions: the *metric formalism* in which the connection is the metric connection of  $g_{ab}$ ; the *Palatini formalism* [5] in which the metric and the connection are independent variables (i.e., the connection  $\Gamma_{bc}^a$  is not the metric connection of  $g_{ab}$ ), the Ricci ten-

sor  $\mathcal{R}_{ab}$  is built out of this connection, and  $\mathcal{R} \equiv g^{ab}\mathcal{R}_{ab}$  is the Ricci curvature appearing in the action. The latter should properly be written as

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(\mathcal{R}) + S^{(m)}[g_{ab}, \psi]. \quad (1.2)$$

In the Palatini formalism, the matter action  $S^{(m)}$  is independent of the (non-metric) connection  $\Gamma_{bc}^a$ ; a third version of  $f(R)$  gravity, the *metric-affine formalism*, allows  $S^{(m)}$  to depend explicitly on this connection. This version is little studied [6] and will not be considered here. We focus on metric and Palatini  $f(R)$  gravity, which give rise to fourth order (in the metric) and second order field equations, respectively. The field equations of metric  $f(R)$  gravity are

$$f'(R)\mathcal{R}_{ab} - \frac{1}{2}f(R)g_{ab} - (\nabla_a \nabla_b - g_{ab}\square)f'(R) = \kappa T_{ab}, \quad (1.3)$$

where a prime denotes differentiation with respect to  $R$  and  $T_{ab} = \frac{-2}{\sqrt{-g}} \frac{\delta S^{(m)}}{\delta g^{ab}}$ . The Palatini field equations are

$$f'(\mathcal{R})\mathcal{R}_{ab} - \frac{1}{2}f(\mathcal{R})g_{ab} = \kappa T_{ab}, \quad (1.4)$$

$$\bar{\nabla}_\lambda [\sqrt{-g} f'(\mathcal{R}) g^{ab}] = 0,$$

where  $\bar{\nabla}_c$  denotes the covariant derivative operator of the independent connection  $\Gamma_{bc}^a$ .

Recently, it has been pointed out that Palatini  $f(R)$  gravity is not viable because of two serious shortcomings:

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(a) When trying to build a stellar model in the weak-field limit using very reasonable (polytropic) fluids, a singularity in the curvature invariants appears at the star's surface, which is related to the impossibility of matching interior and exterior (vacuum) solutions. This feature has been traced back to the fact that the metric depends on derivatives of order higher than first of the matter fields entering the field equations. As a result, discontinuities in the matter distribution are not smoothed out by an integral, as in conventional theories, but the metric depends on the matter fields and their derivatives, causing singularities in the curvature that are physically unacceptable [7].

(b) The initial value problem is not well-formulated nor well-posed for Palatini  $f(R)$  gravity [8]. Nevertheless, many papers still appear on Palatini  $f(R)$  gravity, and here we try to understand its very special features from a completely different perspective.

It is well known that, when  $f''(R) \neq 0$ , metric  $f(R)$  gravity is dynamically equivalent to an  $\omega = 0$  Brans–Dicke (hereafter BD) theory [9] for the massive scalar degree of freedom  $\phi \equiv f'(R)$ , while Palatini  $f(R)$  gravity is equivalent to an  $\omega = -3/2$  BD theory [10]. In the metric formalism, by introducing an auxiliary field  $\chi$ , one can consider the action (dynamically equivalent to (1.1) if  $f'' \neq 0$ )

$$S_0 = \frac{1}{2\kappa} \int d^4x \sqrt{-g} [f(\chi) + f'(\chi)(R - \chi)] + S^{(m)}[g_{ab}, \psi], \quad (1.5)$$

the variation of which with respect to  $\chi$  yields  $f''(R)(\chi - R) = 0$ . By defining the scalar  $\phi \equiv f'(\chi)$ , this action becomes

$$S_0 = \frac{1}{2\kappa} \int d^4x \sqrt{-g} [\phi R - V(\phi)] + S^{(m)}, \quad (1.6)$$

where  $V(\phi) = \chi(\phi)\phi - f(\chi(\phi))$ . This is a BD action with Brans–Dicke parameter  $\omega = 0$  and potential  $V$ . Similarly, in the Palatini formalism, the action (1.2) becomes

$$S_{\text{Palatini}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} [\phi \mathcal{R} - V(\phi)] + S^{(m)}[g_{ab}, \psi], \quad (1.7)$$

but now  $\mathcal{R}$  is not the Ricci curvature  $R$  of the metric connection. The relation between the two is

$$\mathcal{R} = R + \frac{3}{2[f'(\mathcal{R})]^2} [\nabla_a f'(\mathcal{R})][\nabla^a f'(\mathcal{R})] + \frac{3}{f'(\mathcal{R})} \square f'(\mathcal{R}), \quad (1.8)$$

from which one obtains, apart from irrelevant boundary terms,

$$S_{\text{Palatini}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[ \phi R + \frac{3}{2\phi} \nabla^c \phi \nabla_c \phi - V(\phi) \right] + S^{(m)}, \quad (1.9)$$

an  $\omega = -3/2$  BD theory, which is seldom considered in the literature [11]. Note that the Ricci scalar  $R$  appearing in Eq. (1.9) is constructed with the Ricci tensor  $R_{ab}$  of the metric connection of  $g_{ab}$ , and differs from the scalar  $\mathcal{R}$  used earlier.

## 2. Palatini $f(R)$ gravity as a fixed point

The general form of the BD action in the Jordan frame is

$$S_{\text{BD}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[ \phi R - \frac{\omega}{\phi} g^{ab} \nabla_a \phi \nabla_b \phi - V(\phi) \right] + S^{(m)}, \quad (2.1)$$

and the corresponding field equations are

$$R_{ab} - \frac{1}{2} g_{ab} R = \frac{\kappa}{\phi} T_{ab} + \frac{\omega}{\phi^2} \left( \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi \right)$$

$$+ \frac{1}{\phi} (\nabla_a \nabla_b \phi - g_{ab} \square \phi) - \frac{V}{2\phi} g_{ab}, \quad (2.2)$$

$$(3 + 2\omega) \square \phi = \kappa T^{(m)} + \phi \frac{dV}{d\phi} - 2V, \quad (2.3)$$

where  $T$  is the trace of the matter stress–energy tensor. Here we use the equivalence between  $f(R)$  and BD gravities and an invariance property of the latter to elucidate the very special role played by Palatini modified gravity in the broader spectrum of  $f(R)$  and BD theories. Let us consider the gravitational sector of the theory: under the conformal transformation<sup>1</sup>

$$g_{ab} \rightarrow \tilde{g}_{ab} = \Omega^2 g_{ab}, \quad \Omega = \phi^\alpha \quad (\alpha \neq 1/2), \quad (2.4)$$

and the scalar field redefinition

$$\phi \rightarrow \sigma = \phi^{1-2\alpha}, \quad (2.5)$$

and using the transformation property of the Ricci scalar under conformal transformations [4]

$$\tilde{R} = \Omega^{-2} \left( R + \frac{6 \square \Omega}{\Omega} \right), \quad (2.6)$$

the BD action is rewritten as

$$S_{\text{BD}} = \frac{1}{2\kappa} \int d^4x \sqrt{-\tilde{g}} \left[ \sigma \tilde{R} - \frac{\tilde{\omega}}{\sigma} \tilde{g}^{ab} \tilde{\nabla}_a \sigma \tilde{\nabla}_b \sigma - U(\sigma) \right] + S^{(m)}, \quad (2.7)$$

where

$$\tilde{\omega} = \frac{\omega - 6\alpha(\alpha - 1)}{(1 - 2\alpha)^2}, \quad U(\sigma) = V(\sigma^{\frac{1}{1-2\alpha}}), \quad (2.8)$$

i.e., the gravitational part of the BD action is invariant in form under the transformation (2.4) and (2.5). This restricted conformal invariance property is well known and has been likened to the conformal invariance of string theories at high energies [13,14] (remember that the low-energy limit of the bosonic string theory is an  $\omega = -1$  BD theory [15]). The value  $-3/2$  of the parameter  $\omega$  is special; in fact, the function  $\tilde{\omega}(\omega, \alpha)$  is singular at  $\alpha = 1/2$  when  $\omega \neq -3/2$ , has two branches for  $\alpha > 1/2$  and  $\alpha < 1/2$ , and

$$\lim_{\alpha \rightarrow 1/2} \tilde{\omega} = \begin{cases} +\infty & \text{if } \omega > -3/2, \\ -\infty & \text{if } \omega < -3/2, \\ -3/2 & \text{if } \omega = -3/2. \end{cases} \quad (2.9)$$

It is  $\tilde{\omega} < 0$  if  $\omega < -3/2$ ,  $\tilde{\omega} \geq 0$  if  $\omega > -3/2$  and  $\alpha_1 \leq \alpha \leq \alpha_2$ , and  $\tilde{\omega} < 0$  if  $\omega > -3/2$  and  $\alpha < \alpha_1$  or  $\alpha > \alpha_2$ , where  $\alpha_{1,2} = \frac{1}{2}(1 \pm \sqrt{1 + \frac{3\omega}{2}})$ . It is interesting to look for fixed points of the transformation (2.4) and (2.5) as one moves in the space of BD theories ( $g_{ab}^{(\omega)}$ ,  $\phi^{(\omega)}$ ,  $V(\phi)$ ); the theory is already invariant in form under this transformation, and we define as a *fixed point* a BD theory identified by the condition that the BD parameter does not change,  $\tilde{\omega} = \omega$ . The potential  $U(\sigma)$  will, in general, have a different functional form from the potential  $V(\phi)$ , but it seems reasonable to allow for this because the arbitrariness in the choice of the function  $f(R)$  (or  $f(\mathcal{R})$ ) implies arbitrariness in the choice of the potential.

Apart from the trivial cases in which the transformation (2.4), (2.5) reduces to the identity (corresponding to  $\alpha = 0$  or  $\alpha = 1$ ), this equality is satisfied for  $\omega = -3/2$ . Palatini  $f(R)$  gravity, corresponding to an  $\omega = -3/2$  BD theory, is therefore singled out as the unique fixed point of the transformation (2.4), (2.5). It is not difficult to understand why this case is so special: the dynamical field

<sup>1</sup> The transformation (2.4) and (2.5) was used in [12] to study the  $\omega \rightarrow \infty$  limit of BD theory to general relativity, which may fail in the presence of conformally invariant matter.

equation (2.3) for the BD scalar  $\phi$  degenerates into the algebraic identity  $2V - \phi V' = \kappa T$  for this value of  $\omega$ . The dynamical equation for  $\phi$  disappears, leaving this field with no dynamical role.<sup>2</sup> It is exactly this fact that makes the Cauchy problem for Palatini  $f(R)$  gravity ill-formulated and, therefore, ill-posed: because there is no expression for  $\square\phi$  that can be substituted by the matter trace  $T$  in the  $3+1$  decomposition of the field equations, second derivatives of  $\phi$  cannot be eliminated from these equations, contrary to the case  $\omega \neq -3/2$  [8]. The fact that Palatini modified gravity has this (restricted) conformal invariance property singles it out among modified gravity theories, and nowhere is this more evident than in the equivalent BD theory.

An equivalent way of looking at this issue is the consideration of the Einstein frame formulation of this theory. It is well known that, by performing a conformal transformation of the metric and a scalar field redefinition (different from (2.4) and (2.5)), and given instead by

$$g_{ab} \rightarrow \tilde{g}_{ab} = \phi g_{ab} \quad (\Omega = \sqrt{\phi}), \quad (2.10)$$

$$\phi \rightarrow \tilde{\phi} = \int |3 + 2\omega|^{1/2} \frac{d\phi}{\phi}, \quad (2.11)$$

a BD theory is mapped into its Einstein frame representation  $(\tilde{g}_{ab}, \tilde{\phi})$  in which the action (2.1) assumes the form

$$S_{\text{BD}} = \frac{1}{2\kappa} \int d^4x \sqrt{-\tilde{g}} \left[ \tilde{R} - \frac{1}{2} \tilde{g}^{ab} \tilde{\nabla}_a \tilde{\phi} \tilde{\nabla}_b \tilde{\phi} - \frac{V(\tilde{\phi})}{\phi^2} + \mathcal{L}^{(m)}[\phi^{-1} g_{ab}, \psi] \right], \quad (2.12)$$

which corresponds to general relativity with a scalar  $\tilde{\phi}$  minimally coupled to the curvature, but which exhibits an “anomalous” coupling of to the matter fields<sup>3</sup>  $\psi$  (unless the latter are conformally invariant). The Jordan and Einstein frames are physically equivalent representations of the same theory [16,17] (at least at the classical level [18]). The definition of the Einstein frame scalar  $\tilde{\phi}$  breaks down as  $\omega \rightarrow -3/2$ . One can still perform the conformal transformation of the metric without redefining the field  $\phi$ , thus obtaining the Einstein frame action

$$S_{\text{Palatini}} = \frac{1}{2\kappa} \int d^4x \sqrt{-\tilde{g}} \left[ \tilde{R} - \frac{V(\phi)}{\phi^2} + \frac{\mathcal{L}^{(m)}[\phi^{-1} g_{ab}, \psi]}{\phi^2} \right], \quad (2.13)$$

the variation of which with respect to  $\phi$  leads again to  $2V - \phi V' = \kappa T$ . In other words, there is no dynamical equation for the scalar field  $\phi$ , which is appended to the gravitational and matter actions and can be assigned arbitrarily *a priori*. The scalar  $\phi$  becomes unphysical and, therefore, this theory is unattractive from the physical point of view.

In some sense, Palatini  $f(R)$  gravity appears to be unphysical because it corresponds to forcing conformal invariance onto a field that is intrinsically non-dynamical and whose existence is not justified from the physical point of view.  $\phi$  is the inverse of the effective gravitational coupling; in the transformation (2.4), the latter acts as a scaling factor for all lengths, times, and masses. To preserve the *form* of the field equations, this effective coupling itself needs to be changed appropriately (Eq. (2.5)). In the absence of matter and of the potential (or with a quadratic potential), the dynamics of BD theory are left unchanged by these rescalings, at the price of changing the *value* of the BD parameter (Eq. (2.8)).

This parameter, which weights the relative importance of the kinetic and the  $\phi R$  terms in the BD action, can be changed by a large amount. Now, requiring that also the *value* of  $\omega$  be left invariant by the transformation (2.4), (2.5) is simply too much: the only way to achieve this is by losing completely the dynamics of  $\phi$ , which can then be assigned arbitrarily in infinitely many different ways (for example, two prescriptions for  $\phi$  and its derivatives could coincide on an initial hypersurface  $\Sigma$  and differ in the future domain of dependence of  $\Sigma$ , resulting in the non-uniqueness of solutions that makes the Cauchy problem ill-posed [8]). Downgraded to an auxiliary field, the effective gravitational coupling  $\phi^{-1}$  becomes completely arbitrary, which defeats the original purpose of its introduction (i.e., having it determined by the distribution of all masses in the cosmos) [9], and destroys also the more modern motivation of scalar-tensor gravity with  $\phi$  entering as a dynamical dilaton.  $\phi$  is then assigned in a manner external to the theory and cannot be determined internally. In this situation, the only meaningful choice is  $\phi = \text{const}$ , which reproduces general relativity.

It is instructive to examine a toy model in particle dynamics which mimics Palatini  $f(R)$  gravity and was proposed in [19]. This is given by the action

$$S \int dt \left( \frac{wx\dot{Q}^2}{2Q} - xQ\dot{Y} - \frac{x}{2} QY^2 - xJ \right), \quad (2.14)$$

dependent on the parameter  $w$ , where  $x$  corresponds to  $\sqrt{-g}g^{ab}$ ,  $Y$  to the connection  $\Gamma$ , and  $Q$  to  $\phi$ . The analog of the transformation to the Einstein frame is the change of variables  $z \equiv xQ$ ,  $Q \equiv e^{q/\sqrt{w}}$ , which casts the action (2.14) into [19]

$$S = \int dt \mathcal{L} = \int dt \left( \frac{z}{2} \dot{q}^2 - z\dot{Y} - \frac{z}{2} Y^2 - ze^{-q/\sqrt{w}} J \right). \quad (2.15)$$

(A proper treatment should include a Lagrange multiplier: we do not show it here and refer the reader to the discussion of [19].)

What is, in this toy model, the analog of our transformation (2.4) and (2.5)? Ignoring again matter (represented by  $J$ ) for simplicity, it is straightforward to check that this is given by

$$z = xQ^\alpha, \quad Q = q^{\frac{1}{\alpha-1}}, \quad p = 1/q, \quad (2.16)$$

which transforms the Lagrangian density into

$$\mathcal{L} = \frac{\tilde{w} \dot{p}^2}{2p} - zp\dot{Y} - \frac{z}{2} pY^2, \quad \tilde{w} = \frac{w}{(\alpha-1)^2}, \quad (2.17)$$

i.e., the action (2.14) is invariant *in form* under the transformation (2.16). Excluding the trivial situation  $\alpha = 2$  and  $Q = q$ , there is only one occurrence in which also the *value* of the parameter  $w$  is left unchanged, namely the case  $w = 0$ . In this case, the dynamics of the variable  $Q$  are lost. This situation appears rather trivial in the toy model employed, but it shows that imposing too strict of a requirement (the invariance in value of the parameter  $w$  mimicking  $\omega$ ) leads to an unphysical situation. Our main point, that the very special role played by the conformally invariant value  $-3/2$  of the BD parameter  $\omega$  leads to an unphysical theory, is exemplified by the unphysical zero value of the parameter  $w$ . The very special role of these parameter values from the mathematical point of view corresponds to unphysical situations.

To complete the analogy between  $\omega = -3/2$  BD theory and the toy model, one notes that the analog of the transformation to the “Einstein frame” is spoiled when  $w = 0$ . The variable  $Q \equiv e^{q/\sqrt{w}}$  cannot be defined, but the other variable  $z \equiv xQ$  is still well defined and one can define an “Einstein frame” physically equivalent to the “Jordan frame” in terms of  $z$  and  $Q$ . The “Einstein frame” action is (apart from the Lagrange multiplier discussed in [19])

$$S = \int dt \left[ -z \left( \dot{Y} + \frac{Y^2}{2} \right) + Q^{-1} J \right], \quad (2.18)$$

<sup>2</sup> There is an exception: the case in which  $\square\phi = 0$ , which includes general relativity (for  $\phi = \text{const}$ ) and harmonic  $\phi$ -waves.

<sup>3</sup> Usually, only values of the BD parameter  $\omega > -3/2$  are considered in the literature and this transformation appears without the absolute value in the argument of the square root.

where the last term exhibits the non-minimal coupling of the non-dynamical field  $Q$  to matter.

We conclude that the very special value  $-3/2$  of the BD parameter from the mathematical point of view signals the unphysical features of this theory and of Palatini  $f(R)$  gravity. A possible cure (cf., e.g., [7]) would be to generalize the  $f(R)$  action by including terms in  $R_{ab}R^{ab}$ ,  $R_{abcd}R^{abcd}$ , or other invariants of the Riemann tensor. This would have the effect of raising the order of the field equations by two and restore non-trivial dynamics. However, unless the extra terms appear in the Gauss–Bonnet combination, one will be faced with the well-known Ostrogradski instability.

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